MAT: Review: Matrices, Change of Bases, Vietas, Probability and Statistics

1. Add $4_6 + 14_6$. Express your answer in base 6.

2. Find the difference between 1000_7 and 666_7 in base 7.

3. Find the product of $218_9 \cdot 5_9$. Express your answer in base 9.

4. What is the positive difference between the probability of a fair coin landing heads up exactly 2 times out of 3 flips and the probability of a fair coin landing heads up 3 times out of 3 flips? Express your answer as a common fraction.

5. The equations $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r = 0$ have two roots in common. If the third root of each equation is represented by x_1 and x_2 respectively, compute the ordered pair (x_1, x_2).

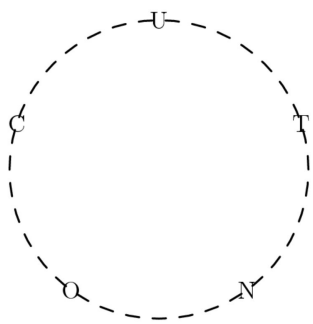
6. Let a, b, and c be the 3 roots of $x^3 - x + 1 = 0$. Find $\frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1}$.

7. Compute $\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}^3$.

8. Given $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, there exist positive real numbers x and y such that

$$(x\mathbf{I} + y\mathbf{A})^2 = \mathbf{A}.$$

9. The letters C, O, U, N and T are randomly placed around a circle. One such arrangement is shown here. If we always start with the C and continue to read the rest of the letters in order clockwise, in how many different orders can the letters appear?



10. What is the base ten equivalent of 101010_5?