MAT: Logarithms

1. What is $\log_7 2400$ rounded to the nearest integer?

2. Evaluate $\log_4 32$.

3. If r, s, and t are constants such that $\frac{x^{r-2}\cdot y^{2s}\cdot z^{3t+1}}{x^{2r}\cdot y^{s-4}\cdot z^{2t-3}} = xyz$ for all non-zero x, y, and z, then solve for $r^s \cdot t$. Express your answer as a fraction.

4. How many distinct four-tuples (a, b, c, d) of rational numbers are there with

$$a \cdot \log_{10} 2 + b \cdot \log_{10} 3 + c \cdot \log_{10} 5 + d \cdot \log_{10} 7 = 2005?$$

5. Solve for the rational numbers x and y:

$$2^{x+y} \cdot 3^{x-y} \cdot 6^{2x+2y} = 72.$$

Express your answer as an ordered pair (x, y).

6. How many integers $-11 \leq n \leq 11$ satisfy $(n-2)(n+4)(n+8) < 0$?

7. Find the sum of all the solutions to

$$(3\log_4(\log_3 x))(2\log_{64}(\log_3 x) - 1) = -1.$$

8. Let x, y, and z be real numbers such that

$$\begin{aligned} \log_2(xyz - 3 + \log_5 x) &= 5, \\ \log_3(xyz - 3 + \log_5 y) &= 4, \\ \log_4(xyz - 3 + \log_5 z) &= 4. \end{aligned}$$

Find xyz.

9. Let $f(x) = \log_b x$, and let $g(x) = x^2 - 4x + 4$. given that $f(g(x)) = g(f(x)) = 0$ has exactly one solution and that $b > 1$, compute b.

10. Let x,y, and z all exceed 1, and let w be a positive number such that $\log_x w = 24$, $\log_y w = 40$, and $\log_{xyz} w = 12$. Find $\log_z w$.